CURRENT TRENDS IN CY MODULI

MINI-COURSES

Tasuki Kinjo

Title: Intrinsic DT theory and Higgs bundles

Abstract: Donaldson–Thomas (DT) theory is a counting theory of coherent sheaves on Calabi-Yau threefolds, and more generally, of objects in 3-Calabi–Yau categories. Recently, DT theory and its categorification have been playing an important role in the study of the topology of the moduli space of objects in 3-Calabi–Yau categories as well as in 2-Calabi–Yau categories through a procedure called dimensional reduction. In particular, DT theory has applications to the study of quiver varieties and the moduli space of Higgs bundles on a curve. This mini-course starts with an introduction to a new framework of DT theory intrinsic to the moduli stack, which we call 'intrinsic DT theory'. This framework enables us to count non-linear objects such as G-bundles and G-Higgs bundles. Using this theory, we will define the BPS cohomology of the moduli space of G-Higgs bundles on a curve, and propose a formulation of the topological mirror symmetry conjecture for semisimple G in the style of Hausel and Thaddeus. I will also explain that an idea from the non-commutative Calabi–Yau geometry can be used to prove a version of the topological mirror symmetry for the moduli space of GL_n-Higgs bundles known as the χ-independence. This mini-course includes several joint works with Chenjing Bu, Ben Davison, Daniel Halpern-Leistner, Naoki Koseki, Naruki Masuda, Andrés Ibáñez Núñez, and Tudor Pădurariu.

Olivier Schiffmann

Title: Cohomological Hecke operators of punctual modifications on smooth surfaces

Abstract: Operators of punctual modifications are an old and classical tool in the study of moduli spaces of vector bundles on curves (as well as in the function field version of the Langlands program). Similar operators have been used by Grojnowski, Nakajima (and then many others) in order to describe the cohomology of Hilbert schemes of points on smooth surfaces. The formalism of cohomological Hall algebras (of zero-dimensional sheaves on a given surface S) allows one to study the algebra of all such operators (independently of the particular choice of moduli space of sheaves on S considered). We will explain how to compute these COHAs for a smooth quasi-projective surface (at least when the cohomology of S is pure) in terms of the so-called $\mathcal{W}_{1+\infty}$-algebra associated to $\mathsf{H}^*(S,\mathbb{Q})$, and indicate some applications. This is joint work with A. Mellit, A. Minets and E. Vasserot (arXiv:2311.13415, *Coherent sheaves on surfaces, COHAs and deformed $\mathcal{W}_{1+\infty}$-algebras*).

Research talks

Merlin Christ

Title: Lax additive constructions for Calabi-Yau structures

Abstract: Relative Calabi-Yau structures are a generalization of the notion of a Calabi-Yau category to functors between categories. On the level of the moduli stack of objects, a relative Calabi-Yau structure induces a shifted Lagrangian structure. In this talk, we will discuss new ways to construct relative Calabi-Yau functors using what are called lax additive constructions. An instance is the gluing of two dg-categories along a bimodule, also called a lax sum. For this, we will introduce the notion of a commutative cube of categories with a Calabi-Yau structure, further generalizing the notion of a relative Calabi-Yau functor. The abstract notions will also be illustrated in concrete examples. For instance, interesting Calabi-Yau cubes arise from anti-canonical normal crossing divisors. This is based on joint work with Tobias Dyckerhoff and Tashi Walde.

Ben Davison

Title: Critical stable envelopes

Abstract: The stable envelope construction was introduced by Maulik and Okounkov, and underpins their approach to producing R matrices for cohomology of Nakajima quiver varieties, and a new class of quantum groups generalising Yangian algebras. The positive halves of these Yangians turn out (via dimensional reduction from 3d to 2d) to be isomorphic to certain examples of critical cohomological Hall algebras. In this talk I will introduce a 3d version (or critical version) of the stable envelope construction, developed in joint work with Tommaso Botta.

Soheyla Feyzbakhsh

Title: Hurwitz-Brill-Noether Theory via K3 Surfaces

Abstract: I will discuss the Brill-Noether theory of a general elliptic K3 surface using wall-crossing with respect to Bridgeland stability conditions. As an application, I will provide an example of a general k-gonal curve from the perspective of Hurwitz-Brill-Noether theory. This is joint work with Gavril Farkas and Andrés Rojas.

Jérémy Guéré

Title: Looking for a quantum Lefschetz formula in higher-genus Gromov-Witten theory

Abstract: Gromov-Witten invariants of a projective variety X come from enumerating curves in X. A famous example for X is the quintic hypersurface in $\mathbb{P}^4$, studied by string theorists regarding mirror symmetry. Invariants from genus zero curves into X have been computed out of invariants of $\mathbb{P}^4$, following the so-called quantum Lefschetz principle. However, invariants from higher genus curves are much harder to compute. In particular, based on a specific example, one may believe that they cannot be deduced from invariants of $\mathbb{P}^4$. In this talk, I will explain my ongoing work in which I am looking for this relation, presenting a conjectural formula, where it comes from and a possible road to proving it.

Francesco Sala

Title: Hecke operators associated to 1-dimensional sheaves

Abstract: The aim of this talk is to introduce a general framework for studying operators in the style of Nakajima–Negut¸ and Hall, extending the classical theory of Hecke operators defined via punctual modifications. In particular, I will present a new algebra of Hecke operators associated

with one-dimensional sheaves on an elliptic curve embedded in a smooth surface. The talk is based on joint work with Diaconescu and Porta (arXiv:2207.08926), as well as with Diaconescu, Porta, and Yu Zhao.

Sarunas Kaubrys

Title: Joyce vertex algebras and coproducts for critical CoHAs of quivers

Abstract: Joyce has defined a structure of (co)vertex algebra on the (co)homology of the moduli of objects of an abelian or derived category. I will recall this construction and explain how we can extend this construction to define a covertex algebra on the critical cohomology of a quiver with potential. I will then explain how this structure is related to the localised coproduct of Davison and explain how the covertex structure is compatible with the Cohomological Hall Algebra structure. If time permits, I will explain two applications, one to cohomological integrality for symmetric quivers and one to deformed Drinfeld coproducts on Yangians of ADE type.

Richard Thomas

Title: A Hopf index for isotropic sections of bundles with quadratic forms

Abstract: The Hopf index theorem counts solutions of equations via a winding number. I'll review this and give 8 equivalent formulations. Then I'll describe 8 analogues for equations which form isotropic sections of bundles with quadratic form. There are applications to cosection localised virtual cycles and to DT^4 virtual cycles, but mainly I'll focus on more elementary topics. This is joint work with Martijn Kool, Jeongseok Oh and Jørgen Rennemo.